Written Exam for the B.Sc. in Economics summer 2012

Microeconomics C

Final Exam

June 6, 2012

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. (20%)

   (a) Let $C > 1$ and consider the following normal-form game:

	A	B
A	$C, -1$	$0, 0$
B	$0, 0$	$1, -C$

   Find the unique mixed Nash equilibrium. In this equilibrium, what is the probability that the outcome is (A, A)? What is the expected payoff for player 1 (the row player)?

   (b) Consider the normal-form game below where player 1 chooses the bi-matrix, player 2 chooses the row, and player 3 chooses the column:

	A_3	B_3
A_2	$1, 5, 5$	$5, 1, 4$
B_2	$0, 4, 2$	$4, 2, 3$

   A_1

	A_3	B_3
A_2	$0, 7, 0$	$6, 2, 1$
B_2	$6, 6, 6$	$4, 1, 4$

   B_1

   Find all pure strategy Nash equilibria.

2. (30%) Consider the following two stage game with three players. In stage one, player 1 chooses among the actions P (for play) and E (for end). If he chooses E then the game ends and each player receives a payoff of 3. If he chooses P the the game continues to stage two where player 2 and 3 play the following simultaneous game (player 2 chooses a row, player three chooses a column):

	L	R
U	$8, 3$	$1, 4$
D	$9, 1$	$0, 0$

   Player 1 receives a payoff equal to the average payoff of player 2 and 3.

   (a) Draw a game tree representing the two stage game. How many subgames are there in the game (excluding the game itself)?

   (b) Find all pure strategy subgame perfect Nash equilibria.

   (c) Find a pure strategy Nash equilibrium that is not subgame perfect.

3. (30%) A rogue state is close to achieving nuclear capability. The countries Y and Z are therefore preparing an attack on its nuclear facilities. The probability that the attack will be a success is

$$p(s_Y, s_Z) = s_Y + s_Z - s_Y s_Z,$$

where $s_i \in [0,1]$ is the share of its military capacity that country i ($i \in \{Y, Z\}$) uses in the attack. If the attack is successful then each country receives a payoff of 1. The cost of participating in the attack for country i is

$$c_i(s_i) = s_i^2.$$

The objective of each country is to maximize its expected payoff from the attack minus the cost.

(a) Suppose the countries Y and Z choose the shares of military capacity to use in the attack simultaneously and independently. Find the Nash equilibrium of this game.

(b) Find the social optimum under the condition that the two countries use the same share of their military capacity. I.e., find the $\bar{s}_Y = \bar{s}_Z = \bar{s}$ that maximizes aggregate payoff from the attack minus costs. Compare with the equilibrium from question (a) and give an intuitive explanation of your findings.

(c) Suppose the game between the two countries is repeated over an infinite time horizon $t = 1, 2, ..., \infty$. The discount factor is $\delta \in (0, 1)$. In this infinitely repeated game, specify trigger strategies such that the outcome of all stages is the social optimum from (b). Find the inequality that must be satisfied for the trigger strategies to constitute a subgame perfect Nash equilibrium (note: you do not have to solve for the smallest value of δ that satisfies the inequality).

4. (20%)

(a) Consider the Spence job-market signaling model from chapter 4 in Gibbons. Suppose we are in the "envy case". Draw a figure illustrating the unique Perfect Bayesian Equilibrium that is consistent with Signaling Requirement 6. Describe briefly the equilibrium and how it relates to the full information outcome.

(b) In the signaling game below, find a separating Perfect Bayesian Equilibrium.